

What You Can—and Can’t—Do with Three-Wave Panel Data *

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Abstract

The recent change of the General Social Survey (GSS) to a rotating panel design is a landmark development for social scientists. Sociological methodologists have argued that fixed-effects (FE) models are generally the best starting point for analyzing panel data because they allow analysts to control for unobserved time-constant heterogeneity. We review these treatments and demonstrate the advantages of FE models in the context of the GSS. We also show, however, that FE models have two rarely tested assumptions that can seriously bias parameter estimates when violated. We provide simple tests for these assumptions. We further demonstrate that FE models are extremely sensitive to the correct specification of temporal lags. We provide a simulation and a proof to show that the use of incorrect lags in FE models can lead to coefficients that are the opposite sign of the true parameter values.

1 Introduction

The recent change of the General Social Survey to a rotating panel design is a landmark development for social scientists. Panel designs have two key advantages over cross-sectional surveys. First, because each respondent appears in the data multiple times, she can in some sense serve as her own “control group,” allowing for more valid causal inferences (Allison 2009:1). Second, because respondents are measured over time, it is sometimes possible to use temporal ordering to ask more complicated questions

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about social processes. But despite the widely acknowledged advantages of panel data, many researchers do not know how to make the most of them. Less-than-optimal uses of panel data are common even in work published in high-prestige outlets (for examples see Halaby 2004).¹ This is perhaps not surprising given (among other reasons) that very few graduate programs in sociology incorporate training on panel data as part of the required curriculum.

The goal of this paper is to provide some practical and theoretical guidance for researchers who have a good grasp of regression but who have limited experience with panel data. In the process, we summarize existing best practices before offering analyses or extensions original to this paper. Because our target audience is not the high-end user but rather the “mid-end” user, we cannot jump right to our original contributions but must first set the stage by outlining what has gone before. Readers already familiar with panel data and fixed-effects models can skip to section 3.

We explore the promise and pitfalls of panel data under two major headings. We first consider the ability of panel data to help control for unobserved heterogeneity. Though this is not recognized in sociology as often as it could be, the primary *raison d'être* of panel data is to help control for unmeasured variables (Halaby 2004:508). We demonstrate how to make the most of these possibilities in the context of the GSS panel while avoiding common pitfalls. We then turn to the use of panel data to establish temporal ordering. Sociologists frequently use the ordering of data to attempt to establish causal order, often using lagged values of key predictors (for recent examples, see Cha 2010; Faris and Felmlee 2011). Though this can be a useful strategy in some circumstances, we show that it can easily lead to misleading results.

We do not attempt to offer a rigorous statistical consideration of the issues involved since there are already many technical treatments (see e.g., Wooldridge 2010). Instead, we aim to distill the recommendations of specialists in these methods, to demonstrate how various models might be useful (or hazardous), and to consider the assumptions of different models from the point of view of our knowledge about the social world. We undertake these goals squarely in the context of the GSS panel and thus do not consider models that might be more appropriate with different data structures (e.g., cross-sectional studies or panels with more than three waves of data).

¹The first author acknowledges that he has not always followed the advice given here.

2 Panel Data and Unobserved Heterogeneity

2.1 Review: Common Models for Panel Data

In the GSS panel, respondents are asked (most) questions three times, once at each wave. Consider a model for the response a respondent might give to a particular question:

$$y_{it} = \mu_t + \mathbf{x}'_{it}\boldsymbol{\beta} + \mathbf{z}'_i\boldsymbol{\gamma} + v_i + \epsilon_{it}, \quad i = 1 \dots N, t = 1, 2, 3 \quad (1)$$

Equation 1 asserts that the answer respondent i gives to question y at time t is a function of five things: what’s going on the world at that time that affects everyone equally (the intercepts μ_t), the values of any observed time-*varying* variables for the respondent, like age or income (the \mathbf{x}_{it} variables), the values of any observed time-*constant* variables for the respondent, like gender or race (\mathbf{z}_i variables), some *unobserved* time-constant, person-specific “stuff” (like personality) that affects the respondents answers equally at all three waves (v_i), and some other idiosyncratic “stuff” that varies from wave to wave for each respondent (ϵ_{it}).² As always in regression, the main threats to causal inference come from unobserved variables, here v_i and ϵ_{it} . If the observed \mathbf{x} and \mathbf{z} variables are correlated with these unobserved factors, then estimates of their effects will be biased.

There are three common ways of handling these sorts of data. The first, typical in sociology (especially with two waves of data), is the lagged dependent variable (LDV) model (Halaby 2004:535). The LDV model takes the following form, where $t = 2$ refers to the second of two waves of data:

$$y_{it} = \mu + \rho y_{t-1} + \mathbf{x}'_{it}\boldsymbol{\beta} + \mathbf{z}'_i\boldsymbol{\gamma} + \epsilon_{it}, \quad t = 2 \quad (2)$$

The idea behind this model (though not always articulated) is that the lagged value of y will serve as a proxy for v , the unobserved between-person heterogeneity that appears in equation 1.³ One hopes that controlling for it will allow for less biased estimates of the effects of the measured predictors (see Morgan and Winship 2007:179-181). The primary shortcoming of this strategy is that it does not take full advantage of the panel data structure, relying on unclear assumptions about the relationship between y_{t-1} and v

²Unless otherwise specified, i always refers to respondents and $t = 1, 2, 3$.

³Sometimes lagged values of y are included because the researcher posits an actual effect of y_{t-1} on y_t (see Halaby 2004:536; Wooldridge 2010:371). Without also modeling unobserved heterogeneity, however, it is impossible to distinguish an autoregressive process like this from the existence of unobserved factors that affect both y_{t-1} and y_t .

instead of attempting to model v directly. We will show below how this method can yield less-than-optimal results.

The other two most common ways of dealing with panel data are random-effects (RE) models and fixed-effects (FE) models. These are unfortunate names, however, since they do not convey the real differences between them (see Wooldridge 2010:285-286). Both FE and RE use the repeated measures of the outcome offered by panel data to estimate the v_i . The only difference between RE and FE lies in the assumption they make about the relationship between v and the observed predictors: *RE models assume that the observed predictors in the model are not correlated with v while FE models allow them to be correlated.* A moment’s reflection on what v represents—all unmeasured time-constant factors about the respondent—should lead anyone to realize that the RE assumption is heroic in social research, to say the least. The idea that the characteristics we don’t (or can’t) measure (like personality or genetic influences) are uncorrelated with the things we usually do measure (like income or church attendance) is implausible.⁴

FE models avoid the RE assumption by using only within-respondent variation to estimate the x -coefficients. In essence, FE models “subtract off” both observed and unobserved time-constant factors using the panel structure of the data. There are a few ways to do this in practice. For simplicity, assume we have only one time-varying predictor whose effect we want to estimate. The most straightforward approach in an OLS context is just to give each respondent his own intercept. This is shown in equation 3, where v_i stands for a dummy variable added to the model for each respondent.⁵

$$y_{it} = \mu_t + \beta x_{it} + v_i + \epsilon_{it} \quad (3)$$

Another strategy is mean differencing (equation 4), which gives the same estimates for β ⁶:

$$y_{it} - \bar{y}_i = (\mu_t - \bar{\mu}) + \beta(x_{it} - \bar{x}_i) + (\epsilon_i - \bar{\epsilon}_i) \quad (4)$$

A final, slightly different, FE model is the change score, or first-difference (FD) model (equation 5), which is like the mean difference model except it

⁴Of course, any time we estimate a cross-sectional regression, we are making the same assumption. The difference with panel data is that we can test this assumption or even avoid it entirely.

⁵The dummy variable method does not work properly for most limited dependent variables (see Allison 2009:16-18, 32-33).

⁶Manually differencing will not, however, produce the correct standard errors. See Allison 2009:18)

subtracts off the respondent's prior value rather than his overall mean.⁷

$$y_{it} - y_{it-1} = (\mu_t - \mu_{t-1}) + \beta(x_{it} - x_{it-1}) + (\epsilon_i - \epsilon_{it-1}) \quad (5)$$

Regardless of the exact estimation strategy, however, the strength of the FE approach is that it controls for everything specific about a respondent that does not vary by time and that has constant effects on the outcome, including rarely measured factors like personality or genetics.

But there are two costs. First, as we noted, FE models use only within-respondent variation to estimate parameters. If a given respondent doesn't, for instance, have any variability in church attendance from wave to wave, then she contributes nothing to estimating the effect of church attendance on any outcomes. Because they rely on less variation, parameter estimates from FE models typically have larger standard errors. But many scholars have argued that this is a small price to pay to avoid the RE assumption if it is incorrect (which it almost always is; see Halaby 2004:527).

The second cost of using FE models is that one cannot get estimates of the effects of time-constant predictors. That is, since FE models use only within-respondent variation, it's impossible to estimate the effects of things that don't vary within respondents. For sociologists, this is a seemingly high cost, since many of the things we're interested in (such as race, gender, or family background) don't change over a four-year period, if ever. This is surely one main reason that sociologists have been slower to adopt FE methods than economists or other social scientists.

Fortunately, it's quite easy to get around the latter objection. There are models that allow combining FE estimates of the effects of time-varying variables with RE-type estimates of the effects of time-constant factors. The most straightforward of these is Allison's hybrid model (Allison 2009:23-25). If we have one time-varying variable (e.g., work hours) and one time-constant variable (e.g., gender), Allison's model is:

$$y_{it} = \mu_t + \beta(x_{it} - \bar{x}_i) + \omega\bar{x}_i + \gamma z_i + v_i + \epsilon_{it} \quad (6)$$

This model is essentially a RE model with a FE twist. Just like a RE model, it assumes that v is uncorrelated with the predictors, but it differs by modeling the time-varying and time-constant parts of x separately. This makes the estimate of β the same as all other FE models while allowing for the inclusion of time-constant variables.

⁷The estimate of β can be somewhat different because the FD model allows adjacent errors to be correlated. See Wooldridge (2010:321-326) for more on the differences between FE and FD models.

2.2 Two Illustrations

Regardless of the exact model used, the consensus among the experts is quite strong that the FE model should be preferred over other approaches (see e.g., Halaby 2004:517-522; Allison 2009:3). But what is the cost of using suboptimal methods? To make these points concrete, we provide two illustrations. We first present the results of a simulation that demonstrates how coefficient estimates can be biased as the correlation between the observed variables and v increases. Next we conduct a simple analysis of GSS data to illustrate the point in a more realistic research context.

2.2.1 Simulation

We simulated three-wave panel datasets that make y a function of x and v . We varied the correlation between x and v from 0 (no confounding time-constant unobserved heterogeneity) up to .9 (a huge degree of confounding time-constant unobserved heterogeneity). The simulation is constructed so that the true $\beta = .4$. For each dataset, we estimate β using five different models: OLS, the LDV model from equation 2, RE, FE, and Allison’s hybrid model. (See the appendix for full details.)

Figure 1 shows that the FE and hybrid models are unaffected by the degree of correlation between the observed and unobserved variables. OLS does the worst, followed by RE. The LDV model does better than either OLS or RE, but only produces unbiased estimates by coincidence when the amount of unobserved time-constant heterogeneity is just right. If the goal is estimating the effect of a time-varying variable (like church attendance, income, or employment status) the FE (or hybrid) estimators have a clear advantage because they purge out all time-constant unobserved factors.

2.2.2 GSS Example

To illustrate these processes further, we consider a (simplified) example from the 2006-2010 GSS panel. Say we are interested in the effect of church attendance, measured on a 0-8 scale, on opposition to abortion rights, measured from 0-6 (see Hout 1999 for more on the scale). We also think the following time-constant variables (measured in 2006) might be relevant: gender, parents’ education (an indicator coded 1 if either parent had at least a BA), and race (indicators for black and other). As before, we compare the results from a pooled OLS model, a lagged dependent variable (LDV) model, a RE model, a FE model, and Allison’s hybrid model. Table 1 presents the estimates provided by all of these models.

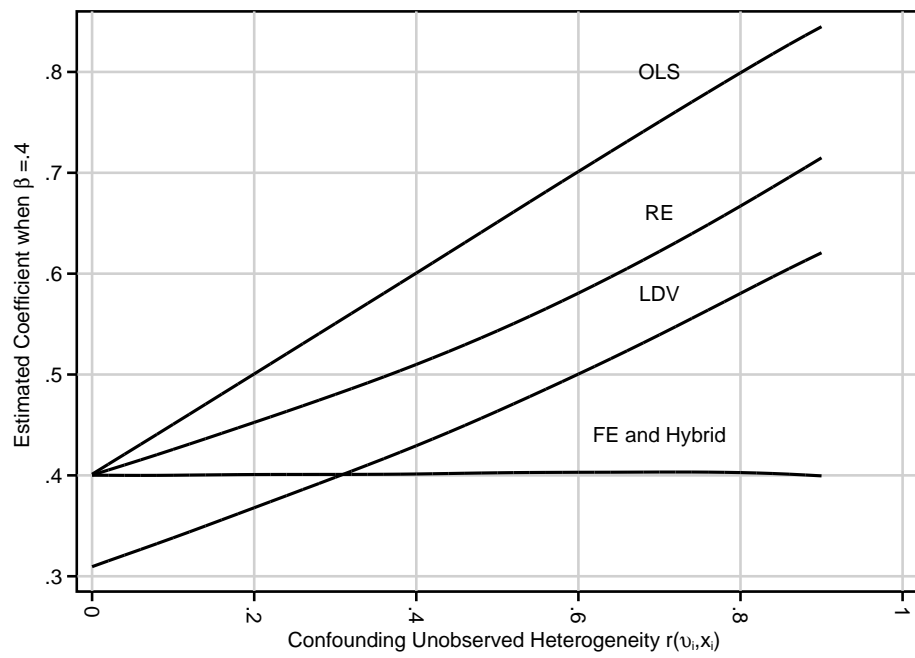


Figure 1: Simulation Results for Different Panel Models

Table 1: Unstandardized Coefficients Predicting Opposition to Abortion

	(1)	(2)	(3)	(4)	(5)
	OLS	LDV	RE	FE	Hybrid
y_{t-1}		.70 [.66,.74]			
Attendance	.31 [.27,.34]	.11 [.08,.13]	.18 [.14,.21]	.055 [.016,.094]	
Δ -Attend.					.055 [.016,.094]
Mean Attend.					.36 [.31,.40]
Female	-.0011 [-.23,.23]	-.056 [-.17,.062]	.13 [-.11,.37]		-.052 [-.28,.18]
Parent BA	-.38 [-.65,-.12]	-.082 [-.21,.041]	-.36 [-.63,-.08]		-.39 [-.66,-.13]
Race: Black	-.19 [-.54,.16]	-.18 [-.37,.0063]	-.054 [-.4,.29]		-.24 [-.59,.11]
Race: Other	.016 [-.33,.36]	-.01 [-.2,.18]	-.051 [-.4,.3]		.041 [-.31,.39]
2008	-.094 [-.2,.013]		-.081 [-.18,.022]	-.069 [-.17,.033]	-.069 [-.17,.033]
2010	-.015 [-.13,.098]	.11 [-.04,.25]	-.013 [-.12,.095]	-.011 [-.12,.095]	-.011 [-.12,.095]
Constant	1.3 [1.1,1.6]	.32 [.17,.46]	1.7 [1.5,2.0]	2.2 [2.0,2.3]	1.2 [.91,1.4]
N	2550	1700	2550	2550	2550

95% confidence intervals in brackets

We begin with estimators that do not model v directly. The OLS estimate of the effect of attendance is .31. This does nothing to control for unobserved heterogeneity and is therefore biased if there is an association between attendance and unmeasured factors related to the outcome. The LDV model adds the value of y from the previous wave in an attempt to model unobserved heterogeneity through its association with v . The estimate is reduced to .11 and the confidence interval now includes 0.

We now turn to the panel models proper. The RE coefficient is .18, between the OLS and LDV estimates. RE models assume that unobserved heterogeneity is not correlated with the observed predictors, but because this is probably violated, the estimate is likely upwardly biased. The FE coefficient in the next column tends to confirm this suspicion. Using only within-respondent variation across waves, the FE model yields a coefficient only about one-third as large (.055), though the confidence interval still does not include zero.⁸ This coefficient is likely a better estimate of the actual *effect* of attendance on attitudes since it is uncontaminated by any time-constant factors with constant effects on abortion attitudes. Note, however, that there are no coefficients for time-constant variables because they do not vary wave to wave.

The final model is Allison’s hybrid model. As must be the case, its estimate of the effect of within-respondent change in attendance (Δ -attendance) is exactly the same as the FE estimate. Here, however, there are coefficients for all time-constant variables (including mean attendance), though they must be interpreted with caution because they are subject to standard concerns about associations with unmeasured factors.

3 Testing the Limits of FE Models

The illustrations above demonstrate what is already widely known: FE methods can offer good protection against bias due to unobserved time-constant heterogeneity. If one must have a “default” model for panel data, FE methods are a good place to start and have definite advantages over the more common (in sociology) RE and LDV models.

FE methods, however, rely on two assumptions that are generally not tested. The first is that selection into levels of x is based on unobserved factors (v) rather than on previous values of y . The second is that the

⁸If the RE assumption were justified, the RE and FE coefficients on attendance would be about the same. This can formally be tested with a Hausman test or other similar tests. See Allison (2009:21-23) and Cameron and Trivedi (2010:266-268).

underlying time trajectories of y are the same regardless of the values x takes (see Morgan and Winship 2007:262-271). By construction, our simulations in the previous section were consistent with these assumptions but real data might not be.

Morgan and Winship (2007, chapter 9) explain how to test for (and in some cases, deal with) violations of these assumptions (see also Elwert and Winship, in progress). But they develop these ideas in the context of binary treatments given to some at a single time point between waves 2 and 3 of a three-wave panel, a situation that will not correspond to many panel analyses with the GSS, where the predictors of interest might vary from wave to wave along with the outcomes. In this section, we extend Morgan and Winship’s ideas to apply to continuous treatment variables whose values can vary from wave to wave.

3.1 Treatment Selection Assumption

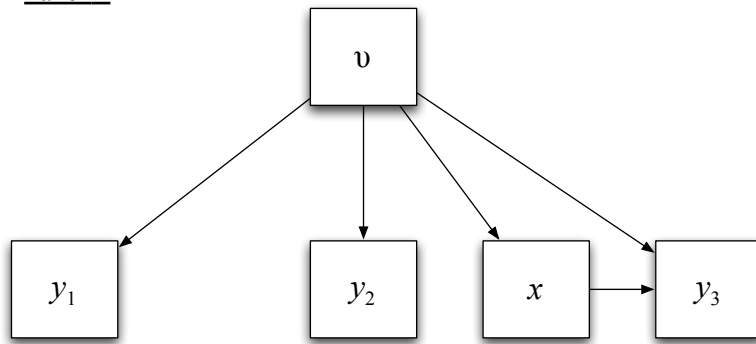
For clarity, we begin with the case of a binary treatment received (or not) between waves 2 and 3 of a three-wave panel. Figure 2 presents these ideas in graphical form. In panel A, we have the classic FE case: y_t are functions of an unobserved time-constant fixed effect (v), selection into the treatment (x) is based on v , and y_3 is affected by both v and x . In such cases, a FE model will work well because it will appropriately separate the effect of v from the effect of x on y_3 .

Panel B changes things slightly by making the treatment, x , a function of both v and y_2 , the previous wave’s outcome variable. It is not difficult to imagine situations like these; (un)happiness may be both a cause and a consequence of divorce, for example. When the data are generated in this manner, FE models will not give unbiased estimates of the effect of x on y_3 because controlling for v alone does not prevent the effect of y_2 on y_3 through x from “leaking through” into the estimate of the effect of x .

We now extend these considerations to multiple continuous “treatments,” such as those measured in surveys like the GSS panel. Church attendance, for example, is a sort of treatment than can vary from wave to wave in different amounts rather than being switched on for some as a binary treatment.

Figure 3 illustrates this extension using two diagrams analogous to their counterparts in Figure 2. If the data were generated as in panel A, FE models will work well; if they were generated as in panel B, FE models will give biased estimates because controlling for v alone does not unconfound the relationship between x_t and y_t .

Panel A



Panel B

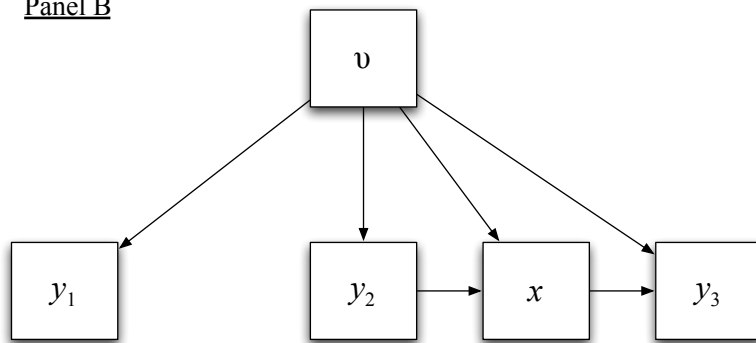


Figure 2: Causal Models of a Single Treatment Without (A) and With (B) Endogenous Selection

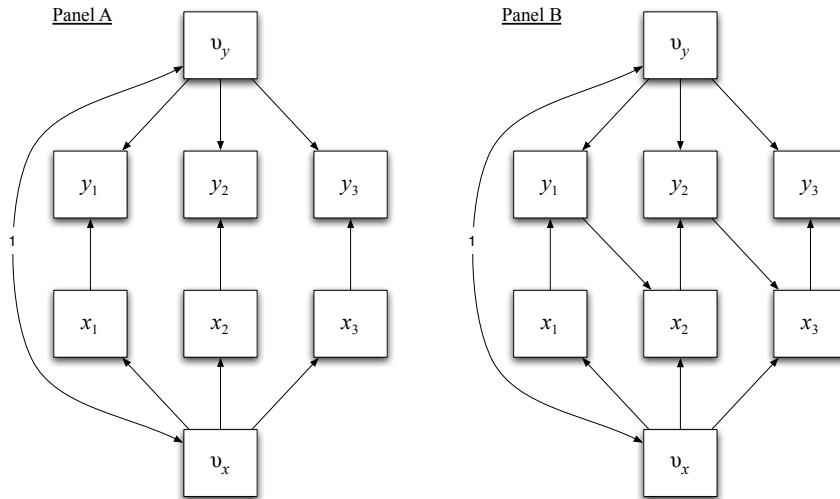


Figure 3: Causal Models of an Ongoing Treatment Without (A) and With (B) Endogenous Selection

3.1.1 Simulation

To demonstrate this point, Figure 4 presents simulations like those we presented earlier. This time, instead of varying the amount of unobserved heterogeneity (which we keep constant at $r[v_i, x_i] = .5$), we vary the extent to which the predictor of interest, x_t , is a function of y_{t-1} , the outcome at the prior wave. We hold the value of β (the effect of x_t on y_t) constant at .4.

There are a variety of biases evident in this figure. As in Figure 1, OLS and RE generally overestimate the effects of x and this bias gets worse when high levels of y_{t-1} lead to higher levels of x_t . The LDV results are more robust to endogenous selection but they are biased because they do not properly account for unobserved factors. Finally, FE and hybrid models quickly become biased in the presence of either positive or negative selection. It is clear at a glance that none of these models is a panacea for both unobserved heterogeneity and endogenous selection into treatment.

There are models that *can* accommodate both time-constant unobserved heterogeneity and endogenous selection but they need to be estimated using structural equation models (SEMs; see Bollen and Brand 2010). SEMs are getting easier to estimate in standard software packages but they are still

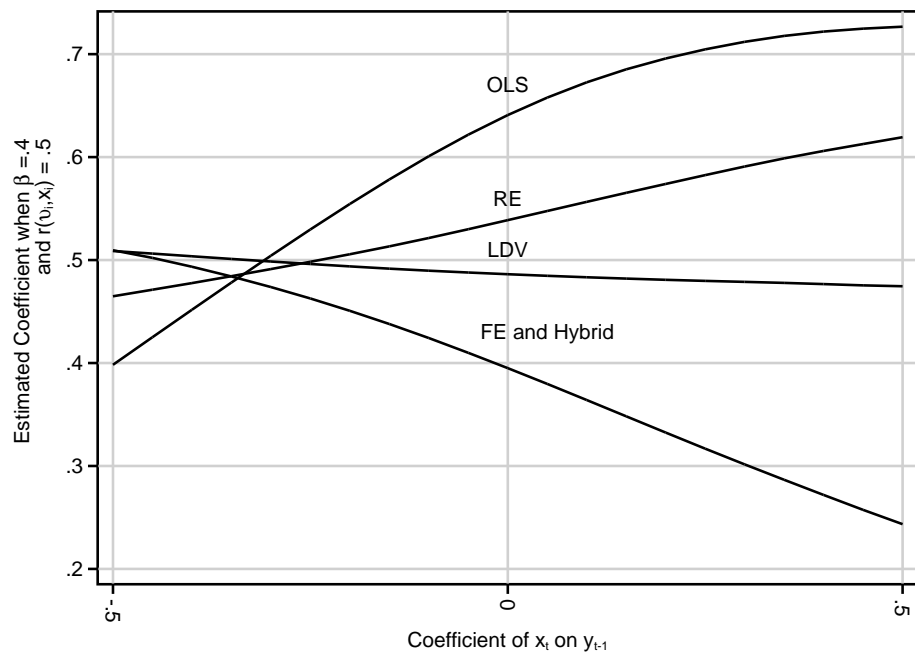


Figure 4: Simulation Results under Varying Levels of Endogenous Selection

uncommon in sociology. In many cases, what we really want to know is whether we *need* to use something more complicated or whether a simpler model is adequate.

3.1.2 Testing the Assumption

Morgan and Winship (2007:267) point out that with two pre-treatment waves of data, we can test the no-endogenous-selection assumption simply and directly. Their proposed test begins with estimating the following model:

$$\log \left(\frac{Pr[x = 1]}{Pr[x = 0]} \right) = a + y_2b + (y_1 + y_2)c \quad (7)$$

The test for endogeneity is the test of $b = 0$. The logic is that if selection into the treatment (x) is a function of the unobserved fixed effect (v) and *not* of the previous wave's outcome, y_2 will have no independent predictive power net of the more informative proxy for the FE ($y_1 + y_2$). If the data are not consistent with $b = 0$, more complex models must be explored.

As before, we extend the Morgan and Winship approach beyond a single binary treatment. With the GSS panel and similar datasets, we may have three treatments with more continuous distributions. In such situations, we do not have two waves of pre-treatment data for *all* treatments but we do have it for the treatment at wave 3. This allows testing whether x_3 can be predicted by the previous wave's outcome (y_2) net of the proxy for the fixed effect ($y_1 + y_2$). Strictly speaking, we are only testing for endogeneity at wave 3, but if we assume the test would be similar if applied to previous waves (a reasonable assumption), we can consider it an overall test of treatment endogeneity.

3.1.3 GSS Example

Returning to our GSS example, we test whether the treatment (church attendance) at wave 3 is associated with wave 2 opposition to abortion (*abscale*) more than with the time-constant FE (proxied by the sum of opposition at waves 1 and 2). Since church attendance is not binary but has 9 possible responses, it is reasonable to use OLS to estimate an analog to equation 7. Specifically, we estimate:

$$attend_3 = a + abscale_2b + (abscale_1 + abscale_2)c + e \quad (8)$$

The estimate of b here is -.12 [-.35,.11] and the estimate of c is .36 [.24,.48]. This means that although opposition to abortion *in general* appears to be associated with church attendance (as indicated by c), there is

little evidence of an independent association with *recent* opposition to abortion. This pattern is not consistent with endogenous selection and therefore this assumption of the FE model appears reasonable. Had b been significantly different from 0, we would have had to consider SEMs if we wanted to deal with unobserved heterogeneity and treatment endogeneity at the same time. For users of the GSS panel who want to use FE models, this is a quick and easy test of this assumption.

3.2 Equal Trajectories Assumption

A second assumption that is not often tested in FE models is that “treated” and “untreated” cases have the same underlying time trajectory prior to treatment. Morgan and Winship (2007:263-264) point this out, developing their critique once again in the context of three-wave data with a single treatment received (or not) between waves 2 and 3.

To fix ideas, consider two hypothetical populations: one that will get married between waves 2 and 3 of a three-wave panel and one that won’t. Let us assume that, for various reasons, those who will get married later are increasing in happiness over time and those who will not are not increasing in happiness over time. Figure 5 illustrates these assumptions visually. Applying a FE model to data like these would lead to the erroneous conclusion that marriage causes increased happiness because the model fails to account for time trends that differ between will-be-treated and won’t-be-treated groups prior to the treatment. Morgan and Winship propose a more flexible model that allows different treatment groups to have different time slopes. Specifying time as linear, their model is:

$$y_{it} = \alpha + \beta x_{it} + \omega x_i^* + \gamma T + \gamma'(T \times x_i^*) + \epsilon_{it} \quad (9)$$

where α is the intercept, β is the effect of the treatment (x), ω is the difference in intercepts at the first wave between will-be-treated ($x^* = 1$) and won’t-be-treated ($x^* = 0$) groups, γ is the slope on time (T) for the won’t-be-treated group, γ' is difference in the slope for the will-be-treated group, and ϵ is an error term. In a case like that represented in Figure 5, the model in equation 9 will correctly find that $\beta = 0$, that the treatment has no effect once time is properly taken into account. Unfortunately, few analyses using the GSS panel will take the exact form of a single binary treatment, meaning that Morgan and Winship’s model cannot be directly applied.

Extending the model in equation 9 to the typical GSS case is not obvious because treatments (like level of church attendance) are ongoing rather than

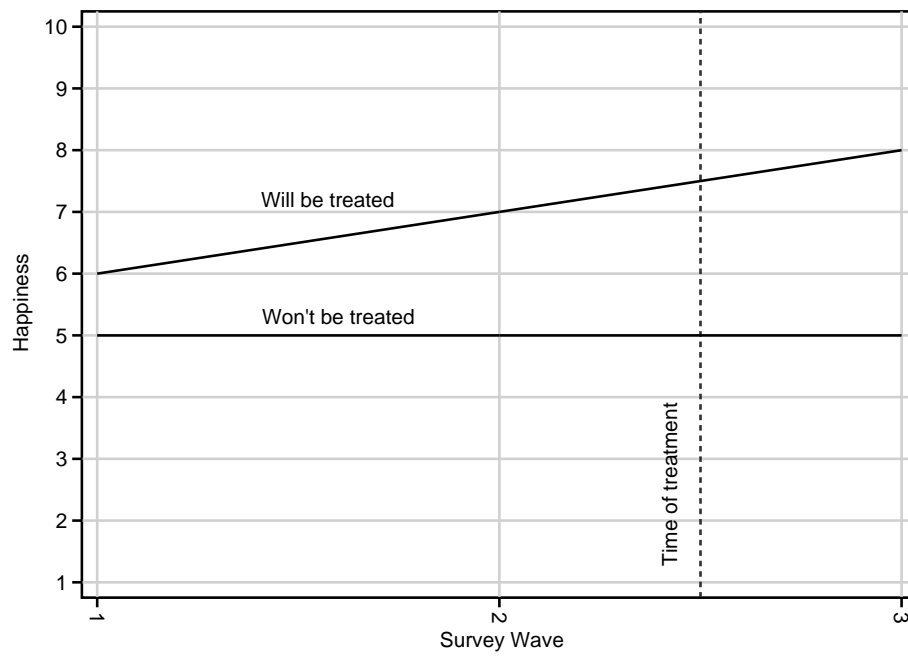


Figure 5: Hypothetical Time Trajectories for Treated and Untreated Groups

taking place at a discrete time in the future.⁹ There is therefore no simple equivalent to x^* , a time-constant indicator of ever-treated status for each respondent.

Using Allison’s hybrid model (see equation 6) as a baseline, we can, however, separate out the time-varying ($x_{it} - \bar{x}_i$) and time-constant (\bar{x}_i) components of x . If we allow \bar{x} to interact with time, we have a model that is very close in spirit to Morgan and Winship’s.¹⁰ Specifically, by estimating

$$y_{it} = \alpha + \beta(x_{it} - \bar{x}_i) + \omega\bar{x}_i + \gamma T + \gamma'(T \times \bar{x}_i) + v_i + \epsilon_{it} \quad (10)$$

we can allow respondents with different *average* levels of x to have different time trajectories. In this model, β will be a FE estimate of the direct effect of x on y that is not biased by potential differences in time slopes for those with different mean values of x .

As with the case of endogenous selection above, we may want to test for these sorts of problems in the hope that using a simpler model is acceptable. Returning to the abortion attitudes and church attendance example, we estimated a hybrid model like that in Table 1, but adding interaction terms between mean attendance and the dummy variables for 2008 and 2010. This specification did not improve model fit ($\chi^2 = 1.14, p = .57$), indicating that the assumption of equal time slopes is reasonable in this particular case.

4 Determining Causal Order

In the last two sections, we argued that panel data can facilitate causal inference by providing leverage on the problem of unobserved heterogeneity. But unobservables are not the only threats to causal inference. Determining causality is a fraught enterprise; scholars have not even reached consensus on its definition, let alone on how to determine it in particular cases (see Morgan and Winship 2007). Bollen (1989:41), however, provides a practical definition that incorporates the major elements shared by most discussions of the issue and that can serve as a useful framework here. First, a cause x must have an *association* with an outcome y . Second, x must be *isolated* from all other causes of y to guard against spurious associations. Finally, x must come before y to establish the *direction* of the effect.¹¹

⁹Some of the ideas in this section emerged from informal discussions with Mike Hout.

¹⁰Here, following Morgan and Winship (2007:269), we specify time linearly. This assumption could easily be relaxed by using dummy variables for survey waves instead of linear time. Indeed, that is what we do in our empirical test.

¹¹This definition is similar to Granger causality (Granger 1969). It is not a philosophical definition of causality, but a practical one.

All statistical models produce conditional associations. FE models add to this a high degree of isolation by removing the effects of time-constant unobserved variables with time-constant effects. The final condition is the establishment of causal direction. In some cases, determining the direction of causality is obvious (income does not affect age, for example). But in other cases, determining causal order is both difficult and of great theoretical or substantive importance. In many sociological subfields some of the most serious debates are about the relative causal priority of two factors that are known to be robustly associated. In the sociology of culture, for example, scholars continue to debate to what extent tastes and worldviews diffuse across existing networks and to what extent cultural similarity leads people to become friends in the first place (see e.g., Lewis, Gonzales, and Kaufman 2012; Lizardo 2006; Vaisey and Lizardo 2010).

Returning to our example from Table 1 (and taking it at face value), we see evidence that there is an association between church attendance and abortion attitudes that is not confounded by any time-constant factors with constant effects. But even in the unlikely event that we have also removed time-varying heterogeneity, the attendance coefficient still represents some unknown mix of the effect of attendance on attitudes *and* the effect of attitudes on attendance. Although either direction of influence is plausible, sociologists generally assume that “structural” factors (like institutional participation) cause “cultural” things like attitudes (see Vaisey and Lizardo 2010). But the models in Table 1 cannot test this assumption empirically because both variables are measured at the same time.

As we mentioned in the introduction, our review of the literature suggests that sociologists regard the establishment of causal order as the prime virtue of panel data. From a theoretical standpoint, it is certainly correct to begin from the premise that causality happens in time. Incantations about “mutual constitution” to the contrary, even reciprocal effects happen in time (see Archer 1995:65-92; Emirbayer and Mische 1998:1002-1003; Vaisey and Lizardo 2010:1611-1612).

By far the most common way for dealing with causal direction in sociology is the use of lagged variables. By predicting the contemporary value of y with the previous waves value of x , the goal is to determine that x precedes y and is associated with it, coming one step closer to a persuasive causal argument. Given the discussion above, we might think that estimating a FE model with lagged predictors would provide the best of all worlds: protection from unobserved heterogeneity *and* the establishment of causal order. Allison (2009:94), who develops a reciprocal effects FE model, extols such models as “enhancing our ability to determine the direction of causality

among variables that are associated with one another.”

The use of models that combine FE and temporal ordering are relatively rare in sociology. The most straightforward such FE estimator is the lagged first-difference (LFD) model:

$$y_{it} - y_{it-1} = (\mu_t - \mu_{t-1}) + \beta(x_{it-1} - x_{it-2}) + (\varepsilon_{it} - \varepsilon_{it-1}), \quad t = 3 \quad (11)$$

In the three-wave case, the LFD estimator models the change in y between waves 2 and 3 with the change in x between waves 1 and 2 (see Martin, van Gunten, and Zablocki [2012] for a recent example). Because it uses differences only, any time-constant unobserved heterogeneity is removed. This model is very clear in its assumptions—the first change is the cause of the second change, and there is no contemporaneous effect of x on y .

It is important to consider the reasonableness of this assumption given the data we are using. In an abstract sense, the t subscript used in all models simply indexes the *ordering* of time. These units of time could be anything from a nanosecond (or less) to a millennium (or more). Compared to data that are widely spaced (as with the GSS panel), short lags will appear practically contemporaneous. This suggests that whether a lagged cause is reasonable given the available data is a matter for theory and substantive knowledge, not for mathematics or statistics (c.f., Martin et al. 2012:33).

For users of the GSS, there is unfortunately no social law stating that a respondents time-varying characteristics from one wave will affect their answers to survey questions two years later. The issue of whether a particular lag is consistent with a theoretically plausible mechanism is one that must be addressed directly. As we show below, getting this wrong can be misleading in the extreme.

4.1 Illustration

Reconsider the example from Table 1. As we mentioned earlier, even in the unlikely event that we have succeeded at purging our FE estimate of all time-variable heterogeneity we cannot know what share of the association is due to the effect of attendance on attitudes and what share is due to the effect of attitudes on attendance because they were measured at the same time. This problem is often resolved by assuming (usually tacitly) that the effects run in the direction hypothesized by the analyst. Instead of making this assumption we might make this an empirical question by regressing the Wave 2 to Wave 3 change in each variable on the Wave 1 to Wave 2 change in the other, as per equation 11. Let us ignore, for the moment, whether this lagged specification is reasonable and estimate the model.

The results are presented in Table 2. The estimated coefficients do nothing to solve the problem that motivated them. Both coefficients are *negative* and the confidence interval of the lagged effect of attitudes on attendance does not include 0. The FE and FD estimates from Table 1 and LFD models here both use the same basic technique for removing unobserved heterogeneity so the difference cannot lie there. The reason for these divergent results must be the different specifications of time.

Table 2: Unstandardized Coefficients from LFD Models

	(1)	(2)
	Attitudes $t3-t2$	Attend $t3-t2$
Attend $t2-t1$	-.038 [-.093,.017]	
Attitudes $t2-t1$		-.09 [-.17,-.013]
N	850	850

95% confidence intervals in brackets

4.2 Simulation

A reasonable question, then, is how the estimates of LFD models are affected by the actual temporal nature of the causal process. By using the lagged difference to predict a subsequent difference, LFD models assume that there is no contemporaneous effect of x on y . Researchers who assume that x and y are not (nearly) contemporaneously related generally do so because they want to use temporal ordering to identify the separate effects of y on x and x on y . Allison (2009:95) reports that LFD models do a good job recovering the correct parameter estimates in his simulations, but these simulations assume that the data were generated by a process with lags matching the spacing of the data collection. In the remainder of this section, we consider how robust LFD models are to violations of this assumption.

To make this question concrete, imagine two worlds, defined by equations 12 and 13:

$$y_{it} = \beta x_{it} + v_i + \varepsilon_{it} \quad (12)$$

$$y_{it} = \beta x_{it-1} + v_i + \varepsilon_{it} \quad (13)$$

In the first world, y is a function of x at the same time point; the lagged value of x has no effect. In the second world, y is a function of x at the

previous time point; the contemporaneous value has no effect. Now we can write an equation that allows for a continuous mixture of these two worlds by adding a new parameter, λ , which can vary from 0 to 1, and where β now represents the total effect of x through both contemporaneous and lagged effects.

$$y_{it} = (1 - \lambda)\beta x_{it} + \lambda\beta x_{it-1} + v_i + \varepsilon_{it} \quad (14)$$

When $\lambda = 0$, equation 14 is identical to equation 12. When $\lambda = 1$, equation 14 is identical to equation 13. When $\lambda = .5$, the contemporaneous and lagged values of x have the same effect on y . As a shorthand where useful, we refer to $\lambda\beta$ as β_{lag} and $(1 - \lambda)\beta$ as β_{con} .

We use a simulation to investigate how the LFD model performs under different mixtures of these causal worlds. Over 500 iterations, each with a “sample” size of 10,000, we allow λ to vary uniformly and allow β to take on three values, .25, .50, and .75. (See the appendix for full details.) We plot the results in Figure 6.

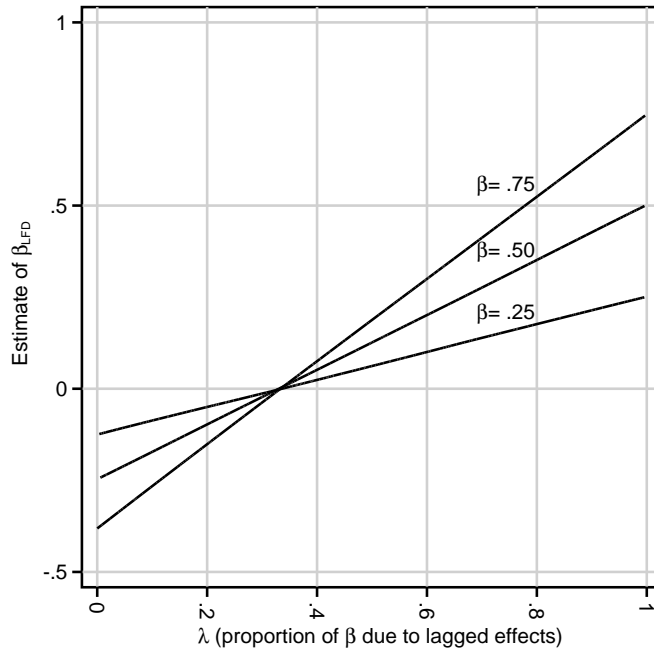


Figure 6: Simulation Results for LFD Models

The pattern that appears in Figure 6 is truly astonishing, but it may take some explaining to see why. It is not surprising that when $\lambda = 1$ (i.e.,

when we are fully in “lag world”) the estimated coefficients are correct. In practice, the LFD estimate of β converges on its true value ($\beta_{lag} = \beta$) when the lags in our data match the causal lags that exist in the real world (as λ approaches 1). But as λ declines toward 0, the estimated value of β_{lag} does *not* decline toward 0 (which is the true effect of x_{t-1} on y_t when $\lambda = 0$), but rather toward $-\frac{1}{2}\beta$.¹² Consider the implication: when x has a causal effect on y that is fully contemporaneous (when $\beta_{con} = \beta$), any lagged- x FE model will yield a coefficient of *opposite sign* and *half the magnitude* of the true causal effect.¹³ We discovered this property through simulation and find it useful to present it that way. But it turns out that, under some very general conditions, this property can be derived analytically (see the appendix for a proof).¹⁴

Because β_{LFD} declines to $-\frac{1}{2}\beta$ when $\lambda = 1$, in order to even hope for a null result, the effect of x_{t-1} on y_t must be at least half as large as the effect of x_t on y_t (i.e., $\lambda = \frac{1}{3}$). In this case, the estimate of β_{LFD} will (within sampling error) be zero *regardless of the size of the total effect of x on y* . Given the two-year lag between panels in the GSS (and almost all panel data studies in sociology), hoping for lagged effects of even that magnitude for most processes is probably too sanguine, meaning that artifactual negative “effects” will likely be the rule rather than the exception.

This is a very surprising—even disturbing—finding. Recall from Table 2 that the LFD estimates of the reciprocal effects of attendance and opposition to abortion were negative. In light of the pattern demonstrated in Figure 2 and our substantive knowledge about religion and U.S. politics, the most reasonable conclusion is that these findings are artifactual. Consider which is more likely: that abortion and church attendance actually have negative reciprocal effects on each other or that two years is too long of a lag to establish causal ordering for this sort of process. Researchers should ask similar

¹²For LFD models, the value is $-\frac{1}{2}\beta$ regardless of the number of waves. For FE models, the value is $-\frac{1}{T-1}\beta$, where t is the number of waves of data. We focus the discussion here on the three-wave case, where this distinction is not relevant.

¹³The opposite holds as well, of course: if $\lambda = 1$, modeling y as a function of contemporaneous x in a FE or FD model will produce a negative artifact in the same manner. We don’t investigate this issue any further here since this world would actually allow us to identify a consistent causal estimate through time ordering (which would be a good thing). Unfortunately, it is hard to imagine many processes in the GSS panel for which $\lambda \approx 1$ is likely to be the case.

¹⁴The basic conditions are that the variance of x and y are the same at each time point and that x is not a state dependent process over the time period of the panel (see Wooldridge 2010:371 for a clear discussion of state dependence). As x becomes fully state-dependent, $\hat{\beta}_{LFD} \rightarrow \lambda\beta$. Our analyses of the GSS panel data (not shown) suggest that very few variables are even weakly state-dependent over the term of the panel.

questions of papers that use such models to find null or “counterintuitive” negative effects.¹⁵

To reiterate, the problem with using lagged variables to establish temporal ordering is that the lags in our data rarely correspond to the lags present in real-world causal processes. Capturing short-term processes with widely-spaced data is best approximated by a cross-sectional approach, but this of course defeats the primary goal that motivated this discussion—the use of temporal ordering to determine causal direction among variables that are known to be associated.

This does not mean that the temporal ordering of the data is useless. For single events that are clearly located in time such as a divorce, job loss, or birth of a child, the ordering of the data can help determine causal effects (with the caveat that one must test the assumptions outlined in Section 3). But for continuously varying states (like attitudes and church attendance), relying on the temporal ordering of the data can be much worse than useless.

5 The Promise and Pitfalls of Panel Data

In recent years, sociological methodologists have encouraged practitioners to rely more on fixed-effects models for analyzing panel data because they are a powerful way of controlling for unobserved heterogeneity (e.g., Halaby 2004; Allison 2009). In section 2 of the paper, we reviewed a variety of common panel models and provided demonstrations of the power of FE models to control for unobserved heterogeneity using both simulated data and GSS panel data.

But as powerful as they are, FE models are not infallible: they rely on two assumptions that are seldom tested and—if violated—can seriously bias causal estimates. In section 3, we built on Morgan and Winship’s discussion

¹⁵As we concluded writing this paper, we came across a paper by Ousey, Wilcox, and Fisher (2011) that uses a lagged predictor SEM model very close to the LFD model used here to estimate the reciprocal relationship between criminal offending and victimization. Despite a long history of research suggesting that these factors are positively related, their model indicates *negative* reciprocal effects using data with a one-year lag. We are by no means willing to assert that their results are artifactual since we know little about the substantive issues involved. This does fit the pattern demonstrated here, however. To be fair, their paper does draw on other research to outline a number of alternative theoretical mechanisms that could account for their findings. England, Allison, and Wu (2007) use the same model with lags ranging from 2 to 9 years. But since their data were on occupational aggregates, this assumption was probably more realistic since institutional change happens on a longer time scale than individual change. This is primarily a matter of theory and substantive knowledge.

of these assumptions, extending their ideas to develop straightforward tests for treatment endogeneity and variable time trajectories in situations similar to those that might be encountered in the GSS panel. These tests are easy to estimate and should become standard practice for researchers who use FE models. If their assumptions are met, FE models can provide powerful protection against unmeasured influences. Otherwise, analysts should explore more flexible alternatives using structural equation models (see Bollen and Brand 2010).

Finally, in section 4, we demonstrated that FE models are extremely sensitive to the correct specification of time, and therefore typically *cannot* be used straightforwardly to settle arguments about causal priority. Our simulations revealed that using lagged regressors in FE models can yield incorrect substantive conclusions when causal lags in the real world do not match the lags found in panel data. In extreme circumstances, estimates will be half the magnitude and in the opposite direction of the true parameter values.

At the risk of oversimplifying, we can summarize our paper in three recommendations to users of the GSS panel and other three-wave panel datasets:

1. Use fixed effect models with panel data to control for time-constant unobserved heterogeneity
2. Test the assumptions of FE models about endogenous selection and temporal trajectories, and use alternative models if these assumptions are violated
3. Do not rely on the ordering of the data to establish causal priority unless the lags between panels match the real world causal lags in the processes under study

As we stated in the introduction, the GSS panel provides new opportunities for social scientists to get more compelling and accurate answers to their research questions. Getting these answers, however, will require users to understand the tools that are at their disposal and apply them appropriately. FE models are a powerful and (we believe) underused method that leverages the power of panel data to provide protection against unobserved heterogeneity, but they are not a panacea. Ultimately, the confidence we place in our estimates must also rely theoretical justifications for the adequacy of our controls for time-varying confounders, and the extent to which our data accurately captures the real-world processes under study.

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Appendix A: Proof

Let

$$X = \epsilon_X, Y = \epsilon_Y$$

Also, let

$$X_1 = X + \epsilon_{X_1}, X_2 = X + \epsilon_{X_2}, X_3 = X + \epsilon_{X_3}$$

and

$$Y_1 = Y + \beta X_1 + \epsilon_{Y_1}, Y_2 = Y + \beta X_2 + \epsilon_{Y_2}, Y_3 = Y + \beta X_3 + \epsilon_{Y_3}$$

Here the ϵ_{2s} represent standard normal i.i.d. normal random variables. Then it follows that

$$(Y_3 - Y_2) = (\beta\epsilon_{X_3} - \beta\epsilon_{X_2}) + (\epsilon_{Y_3} - \epsilon_{Y_2})$$

and

$$X_2 - X_1 = \epsilon_{X_2} - \epsilon_{X_1}$$

Now the regression coefficient for regressing $Y_3 - Y_2$ on $X_2 - X_1$ is equal to

$$\frac{Cov(X_2 - X_1, Y_3 - Y_2)}{Var(X_2 - X_1)}$$

Since the ϵ s are independent,

$$Var(X_2 - X_1) = Var(X_2) + Var(X_1) = 2$$

Also, since the ϵ s are independent with mean 0,

$$\begin{aligned} Cov(X_2 - X_1, Y_3 - Y_2) &= \\ E[(X_2 - X_1)(Y_3 - Y_2)] &= \\ E[(\epsilon_{X_2} - \epsilon_{X_1})(\beta\epsilon_{X_3} - \beta\epsilon_{X_2} + \epsilon_{Y_3} - \epsilon_{Y_2})] &= \\ -\beta E[\epsilon_{X_2}^2] &= \\ -\beta & \end{aligned}$$

Therefore the regression slope is equal to $\frac{-\beta}{2}$, as asserted.¹⁶

¹⁶Thanks to Eric Bair for providing this proof.

Appendix B: Stata Simulation Code

```
*** Model Simulations

quietly {

version 12.1
tempname results
tempfile figure1data
postfile 'results' corr OLS LDV RE FE HYB using 'figure1data'
set seed 092307

local corr = -.02
qui forvalues n = 1/46 {

*** CREATE THE DATA

clear

local corr = 'corr' + .02 // incrementing UH

mat C = (1, 'corr' \ 'corr', 1)
corr2data X Y, n(10000) corr(C)
gen id = _n

local beta = .4 // Beta

gen x1 = X + rnormal()
gen x2 = X + rnormal()
gen x3 = X + rnormal()

gen y1 = Y + rnormal() + 'beta'*x1
gen y2 = Y + rnormal() + 'beta'*x2
gen y3 = Y + rnormal() + 'beta'*x3

egen mx = rowmean(x1-x3)

reshape long y x, i(id) j(wave)
xtset id wave
```

```

gen dx = x - mx

*** MODELS

reg y x
local OLS = _b[x]

reg y L.y x
local LDV = _b[x]

xtreg y x, re
local RE = _b[x]

xtreg y x, fe
local FE = _b[x]

xtreg y dx mx, re
local HYB = _b[dx]

*** POST RESULTS

post 'results' ('corr') ('OLS') ('LDV') ('RE') ('FE') ('HYB')
noi di 'n' // counter

}

postclose 'results'
use 'figure1data', clear

twoway (lowess OLS corr, lw(medthick)) ///
      (lowess LDV corr, lw(medthick)) ///
      (lowess RE corr, lw(medthick)) ///
      (lowess FE corr, lw(medthick)) ///
      /// (lowess HYB corr, lw(medthick)) ///
      , ytitle("Estimated Coefficient when  $\beta = .4$ ") ///
      xtitle("Confounding Unobserved Heterogeneity  $r(\epsilon_i, x_i)$ ") ///
      legend(off) ///
      text(.8 .7 "OLS") ///
      text(.66 .7 "RE") ///
      text(.58 .7 "LDV") ///

```

```

    text(.44 .7 "FE and Hybrid") scheme(rbn1mono)

gr export uhsim.pdf, replace

}

*** Endogenous Selection Figure

quietly {

clear all
version 12.1
tempname results
tempfile figure4data
postfile 'results' endo OLS LDV RE FE using 'figure4data'
set seed 22676

local endo = -.55
qui forvalues n = 1/21 {

*** CREATE THE DATA
clear
mat C = (1, .5 \ .5, 1)
corr2data X Y, n(10000) corr(C)

gen id = _n
local endo = 'endo' + .05

gen y0 = Y + rnormal()

forvalues n = 1/3 {
local t = 'n'-1
gen x'n' = 'endo'*y't' + X + rnormal()
gen y'n' = .4*x'n' + Y + rnormal()
}

reshape long y x, i(id) j(wave)
xtset id wave

```

```

*** MODELS

reg y x
local OLS = _b[x]

reg y L.y x
local LDV = _b[x]

xtreg y x, re
local RE = _b[x]

xtreg y x, fe
local FE = _b[x]

*** POST RESULTS

post 'results' ('endo') ('OLS') ('LDV') ('RE') ('FE')
noi di 'n' // counter

}

postclose 'results'
use 'figure4data', clear

tway (lowess OLS endo)(lowess LDV endo)(lowess RE endo) ///
      (lowess FE endo) ///
, ytitle("Estimated Coefficient when  $\beta = .4$ " ///
"and  $r(\epsilon_{it}, x_{it}) = .5$ ") ///
xtitle("Coefficient of  $x_{it}$  on  $y_{t-1}$ ") ///
legend(off) ///
text(.67 0 "OLS") ///
text(.56 0 "RE") ///
text(.51 0 "LDV") ///
text(.43 .05 "FE and Hybrid") scheme(rbn1mono)

gr export endoselect.pdf, replace

}

```

```

*** LFD Simulation Figure

version 12.1
tempname results
tempfile figure4data
postfile 'results' beta lambda b_lfd using 'figure4data'
set seed 22676

qui forvalues n = 1/500 {

*** CREATE THE DATA

clear
local corr = .5 // level of UH (irrelevant here)
mat C = (1, 'corr' \ 'corr', 1)
corr2data X Y, n(10000) corr(C)
gen id = _n

local beta = (1+int((3-1+1)*runiform()))*.25
local lambda = runiform()

gen x1 = X + rnormal()
gen x2 = X + rnormal()
gen x3 = X + rnormal()

gen y2 = Y + rnormal() + (1-'lambda')*'beta'*x2 + 'lambda'*'beta'*x1
gen y3 = Y + rnormal() + (1-'lambda')*'beta'*x3 + 'lambda'*'beta'*x2

reshape long y x, i(id) j(wave)
xtset id wave

*** MODEL

reg D.y LD.x
local b_lfd = _b[LD.x]

*** POST RESULTS

post 'results' ('beta') ('lambda') ('b_lfd')
noi di 'n' // counter

```



```

}

postclose 'results'
use 'figure4data', clear

tway (lowess b_lfd lambda if beta==.25) ///
(lowess b_lfd lambda if beta==.5)(lowess b_lfd lambda if beta==.75) ///
, aspect(1) ytitle("Estimate of  $\beta_{\text{LFD}}$ ") ///
xtitle(" $\lambda$  (proportion of  $\beta$  due to lagged effects)") ///
legend(off) scheme(rbn1mono) ///
text(.56 .8 " $\beta = .75$ ", place(w)) ///
text(.38 .8 " $\beta = .50$ ", place(w)) ///
text(.21 .8 " $\beta = .25$ ", place(w))

gr export lfdsim.pdf, replace

```